

Binary Black Hole Encounters, Gravitational Bursts and Maximum Final Spin

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The spin of the final black hole in the coalescence of nonspinning black holes is determined by the “residual” orbital angular momentum of the binary. This residual momentum consists of the orbital angular momentum that the binary is not able to shed in the process of merging. We study the angular momentum radiated, the spin of the final black hole and the gravitational bursts in a sequence of equal mass encounters. The initial orbital configurations range from those producing an almost direct infall to others leading to numerous orbits before infall, with multiple bursts of radiation while merging. Our sequence consists of orbits with fixed impact parameter. What varies is the initial linear, or equivalently angular, momentum of the black holes. For this sequence, the final black hole of mass M_h gets a maximum spin parameter $a/M_h \approx 0.823$, with this maximum occurring for initial orbital angular momentum $L/M_h^2 \approx 1.176$.

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A few years ago, after a decades-long period of development, breakthroughs were made in computational modeling of strong gravitational fields that now allow numerical relativists to successfully simulate binary black holes (BBH) from inspiral through merger. In general terms, there are now two computational recipes to follow. One of them is based on a generalized harmonic formulation of the Einstein equations [1, 2] and uses *excision* [3, 4] of the black hole (BH) singularities. The other recipe, called the moving puncture recipe, involves a BSSN [5, 6] formulation, *punctures* to model BH singularities and a gauge condition for these punctures to move throughout the computational domain [7, 8]. Using these recipes, many studies involving interacting BHs and their generated gravitational radiation have been carried out, including gravitational recoil [9, 10, 11, 12, 13, 14, 15], spin hang-up [16] and matches to post-Newtonian (PN) approximations [17, 18]. Most center on astrophysical implications and connection to future gravitational wave observations.

BBH simulations also enable studies of strong nonlinear phenomena regardless of traditional gravitational astrophysics consequences. A recent example is the work in Ref. [19] on the self-similar behavior found in the approach to the merger/flyby threshold of BBHs. Similar merger thresholds in BBH encounters or scatterings form the context for our work.

We consider orbits in which the BHs initially fly past one another, but then fall back to orbit and merge. We focus on the gravitational waveform and the angular momentum radiated from such encounters. Serendipitously, we find significant astrophysical implications, both the existence of a maximum in the final BH spin and of multiple encounter orbits with associated multiple bursts of gravitational radiation. Ref. [19] considered only the first close encounter or “whirl,” and the study did not extend the evolutions to find possible fall-back orbits such as

those here considered. The work in Ref. [19] and our work here have to date been the only studies considering these highly eccentric orbits; while there have been high-order PN studies of inspiral, cases studied so far have described relatively smooth inspirals [20].

All our orbits are parabolic or hyperbolic encounters. Depending on the merger, the fraction of angular momentum radiated varies significantly ($0.05 \lesssim J_{rad}/L \lesssim 0.55$ with L the initial orbital angular momentum of the binary). This emission of angular momentum sets an upper limit of $a/M_h \approx 0.823$ for the spin parameter of the final BH; this maximum occurs when $L/M_h^2 \approx 1.176$, with M_h the mass of the final merged BH.

As in our previous BBH studies [11, 21, 22], we use a code based on the BSSN formulation and the moving puncture recipe. The results here were obtained with a $634^3 M$ computational domain consisting of 10 refinement levels, with finest resolution of $M/52$. We set up nonspinning equal mass BHs using Bowen-York initial data [23]. The mass of each BH is $M/2$, computed from $\sqrt{A_{ah}/16\pi}$ with A_{ah} their apparent horizon area. The data have the BHs on the x -axis: BH_{\pm} is located at $\pm 5 M$ and has linear momentum $\vec{P}_{\pm} = (\mp P \cos \theta, \pm P \sin \theta, 0)$. We keep the angle constant at $\theta = 26.565^\circ = \tan^{-1}(1/2)$; thus the impact parameter is $\sim 4.47 M$. The total initial orbital angular momentum is given by $\vec{L}/M^2 = 10 (P/M) \sin \theta \hat{z}$. We obtain a one-parameter family of initial data by varying the magnitude of the initial momentum in the range $0.1145 \leq P/M \leq 0.3093$. At the lower limit of the momenta, merger occurs within less than half an orbit of inspiral. We then consider successively higher initial momentum until we find solutions that will clearly require a very long, “infinite” time to merge.

The results are summarized in Figs. 1 and 2. The top panel in Fig. 1 shows the spin a/M_h of the final

BH as a function of the initial orbital angular momentum L/M_h^2 . The spin and mass of the final BH were computed using the apparent horizon formula [21, 24]. The bottom panel of Fig. 1 displays the fraction of angular momentum radiated ($J_{rad}/L = 1 - a/M_h/L$). Figure 2 shows, as a function of L/M_h^2 , in the top panel the final mass M_h/M_{adm} relative to the total ADM mass and in the bottom panel the fraction of energy radiated $E_{rad}/M_{adm} = 1 - M_h/M_{adm}$. The vertical lines in Figs. 1 and 2 denote the value of L/M_h^2 where a/M_h is maximum. We have also calculated both the radiated angular momentum and energy via the Weyl tensor. The results are consistent with those in Figs. 1 and 2. However, the values obtained from $J_{rad}/L = 1 - a/M_h/L$ and $E_{rad}/M_{adm} = 1 - M_h/M_{adm}$ are more accurate because they are not as susceptible to resolution effects as those derived from wave extraction. We have carried out simulations at resolutions of $M/45$, $M/48$, $M/52$ and $M/64$ for ten representative cases in Figs. 1 and 2 to check convergence and make error estimates. We found that the results are consistent with the 4th-order accuracy of our code and that the errors in the quantities displayed in these figures are not larger than 3%.

We have selected six encounters that are representative of the different behaviors in our series. These six cases are $L/M^2 = 0.512, 1.208, 1.352, 1.376, 1.382$ and 1.387 or equivalently $L/M_h^2 = 0.521, 1.282, 1.522, 1.480, 1.498$ and 1.554 . We will refer to them as encounters Ea, Eb, Ec, Ed, Ee and Ef, respectively. Cases Ed, Ee and Ef correspond to the last three points in Figs. 1 and 2.

For $L/M_h^2 \lesssim 0.8$, the radiated angular momentum is $J_{rad}/L \lesssim 0.15$, so the final BH has a/M_h close to L/M_h^2 . The evolution is rather simple in these cases: immediate merger, with minimal inspiral. For instance, in case Ea (Fig. 3), $L/M_h^2 = 0.521$, and $J_{rad}/L = 0.05$; thus most of the angular momentum goes into the final BH, $a/M_h = 0.496$. Fig. 4.Ea shows the corresponding radiated gravitational wave ($M r \text{Re} \Psi_4^{2,2}$). All waveforms were extracted at radius $50 M$.

As the initial angular momentum increases, the radiated angular momentum also increases, suppressing and limiting the spin of the final BH. Eventually for large enough initial angular momentum, so much angular momentum is radiated that, as seen in Fig. 1, the final spin reaches a maximum of $a/M_h \approx 0.823$ at $L/M_h^2 \approx 1.176$. Fig. 3.Eb shows the tracks of the BHs in the neighborhood of this maximum. Fig. 4.Eb shows the corresponding radiated waveform. For even larger initial angular momentum, the spin of the final BH actually decreases for increasing L/M_h^2 . The reason is that the merger is not only preceded by several hang-up orbits [16, 19], but also the merger yields a highly distorted BH that radiates copiously as it settles down. Case Ec with $a/M_h \approx 0.68$ and $L/M_h^2 \approx 1.522$ represents this situation in which almost 50% of the initial angular momentum is radiated (see path in Fig. 3.Ec and radiated waveform in Fig. 4.Ec).

A persistent feature of the mergers with $L/M_h^2 \lesssim 1.3$ is that the separation between the BHs (the coordinate

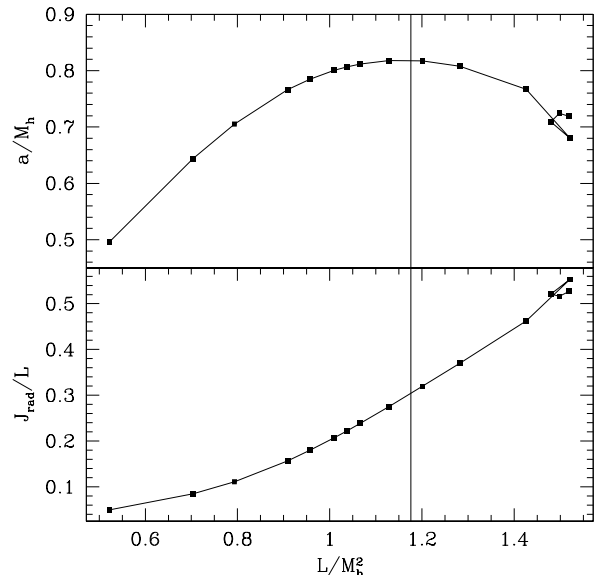


FIG. 1: Top panel, spin of the final BH a/M_h and, bottom panel, angular momentum radiated J_{rad}/L vs the initial orbital angular momentum L/M_h^2 .

distance between the punctures) decreases monotonically with time (monotonic inspiral). Comparing cases Ea, Eb and Ec in Fig. 4, we see general qualitative agreement: inspiral-generated gravitational waves with frequency and amplitude increasing in time, followed by essentially fixed-frequency ringdown waves. There is, however, a hint of disappearance of the monotonic spiral in case Ec. The amplitude of the gravitational radiation has a “shoulder” at about time $\sim 110 M$. For a period of time equal to two wave oscillations, the decline of the amplitude ceases and then recommences. The relative orbital separation as a function of time (Fig. 6.Ec) clearly shows there is a plateau in the separation centered at time $\sim 50 M$, which is absent for cases Ea or Eb. For a brief period of time there is a closely circular phase in which the BHs “want” to fly apart, but just manage to stay at roughly constant separation.

The last three points in Figs. 1 and 2 are the cases labeled Ed, Ee and Ef. They describe orbits without immediate merger but “escape” and recapture; they all show initial approaches followed by increasing mid-evolution separations of $14 M$, $25 M$ and $42 M$ before the final merger (see Figs. 3 and 6). Because the interaction involves two close approaches, there are two bursts of gravitational radiation, one from the first flyby [25] and the other from the merger (see Fig. 5). We are currently investigating astrophysical implications of detections of

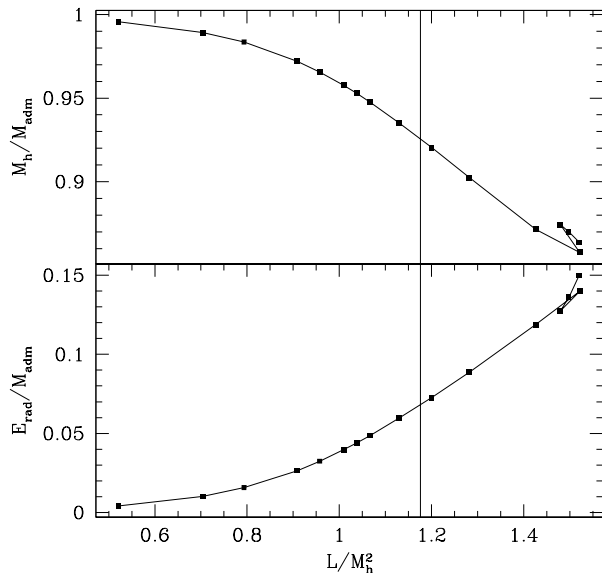


FIG. 2: Top panel, mass of the final BH M_h/M_{adm} and, bottom panel, energy radiated E_{rad}/M_{adm} vs the initial orbital angular momentum L/M_h^2 .

these multiple gravitational bursts and hangups in globular clusters [26].

For the Ef case, there is an approximate hangup with separation $\sim 4 - 5 M$ around time $\sim 950 M$ similar to the shoulder seen in Fig. 6.Ec around time $\sim 50 M$. This structure shows up in the waveform for this Ef case; we actually see a (lower amplitude) precursor to the radiation burst associated with the merger, a hint that orbits with many repeated bounces are possible. For even slightly (0.1%) greater initial angular momentum than case Ef, the BHs complete approximately one loop and then escape. This is a possible indication of chaotic behavior (exponential dependence on initial conditions, c.f. Ref. [19]). Repeated bounce orbits would have to be found with initial angular momentum very slightly above that which resulted in Fig. 3.Ef. As with all critical phenomena, the problem becomes one of careful tuning of the parameters. Note that these interactions of *nonspinning* BHs produce chaotic orbital dynamics, in contrast to the chaos found in *spin* evolutions [27, 28].

One of the main conclusions of our work is that *there is an upper limit on the Kerr parameter* for the final merged BH from nonspinning BH merger. For our sequence this maximum is $a/M_h \approx 0.823$. We can understand this observation by examining the timing of the formation of the final BH and the radiation from the merger. It appears that the merger occurs through an intermediate ex-

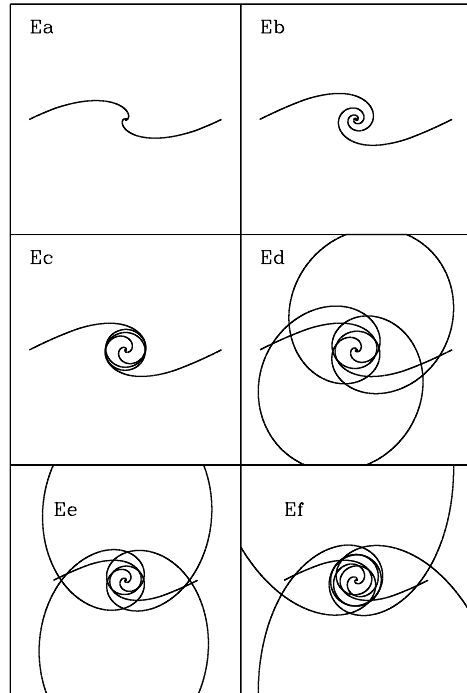


FIG. 3: BH tracks of the encounters. The coordinate dimensions of the top four panels are $12 M \times 12 M$ and $16 M \times 16 M$ for the 2 bottom panels.

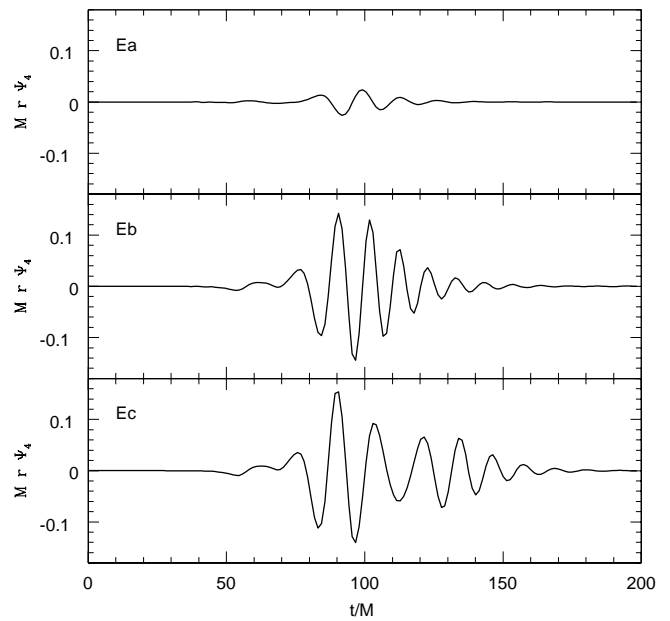


FIG. 4: Waveforms for the Ea, Eb and Ec encounters.

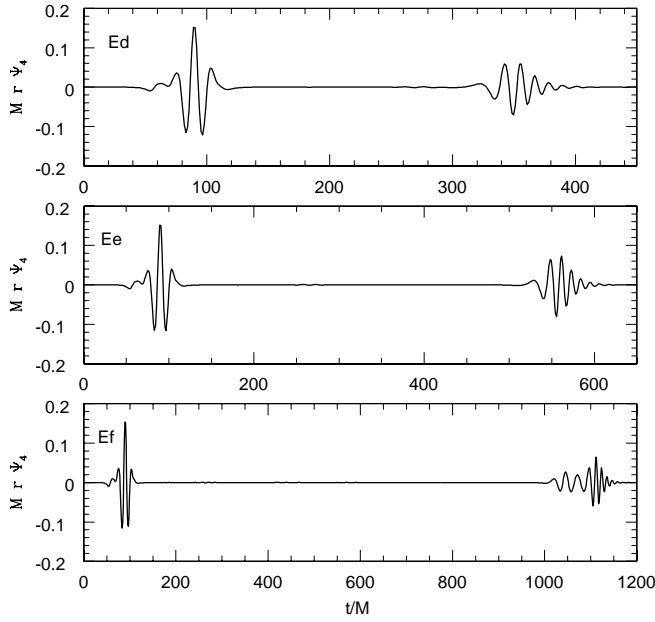


FIG. 5: Waveforms for the Ed, Ee and Ef encounters.

cited state which is essentially a highly distorted BH. We say “essentially” because a substantial amount of angular momentum is also radiated in the plunge immediately before the apparent horizon forms. This is consistent with *close limit* BBH calculations [29] that show merging BHs behaving like a perturbed BH, even before a common apparent horizon forms, so long as the merging BHs are inside the peak of the effective potential of what will be the final BH. This intermediate state emits the largest part of the radiated energy and angular momentum. Because this mechanism is universal (excitation of such a state is inevitable, and it will inevitably radiate), it suggests that *no* merger of equal mass (or presumably, roughly equal mass) BHs can lead to a final BH with maximal spin parameter $a/M_h \approx 1$. This result does not directly affect spin up by accretion since mass accretion will not excite the low l modes that strongly radiate angular momentum. Thus typical gas accretion can in principle lead to final spins much closer to the limit $a/M_h = 1$.

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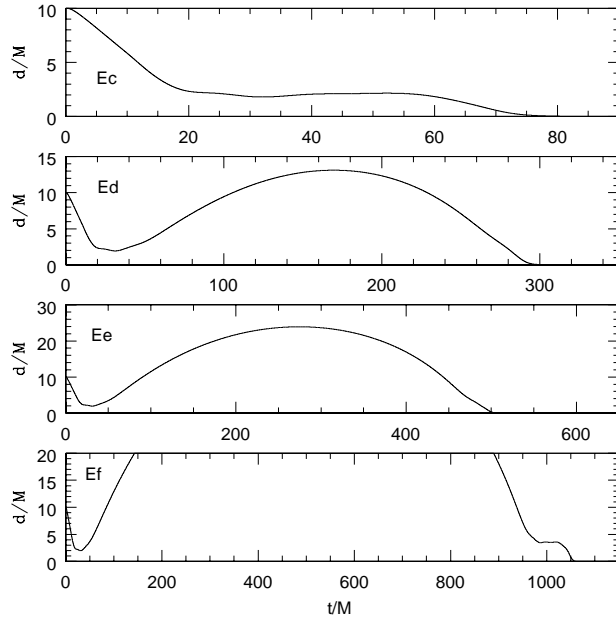


FIG. 6: BBH coordinate separation for the Ec, Ed, Ee and Ef encounters.